Sl-NO of Q.P : 6304 (1)

Unique Paper code: 2221601

Name of the Paper: Solid State Physics (PHHT-13)

Name of the Course: B.Sc. ~~Physics FYUP~~ (Hon) Physics

Semester: -VI ~~Semester~~

Duration: **3 Hours**

Maximum Marks: **75**



F-10

Attempt *Five* questions in all, including Question No. **1** which is compulsory.

*All* questions carry equal marks. Attempt any five of the following:

1. (i) Explain coordination number and atomic radius and calculate them for bcc lattice.

(ii) Discuss the structure of NaCl unit cell.

(iii) What useful information can be obtained from B-H curve of a material?

(iv) Explain why for certain dielectrics, dielectric constant changes abruptly at melting points?

(v) What do Miller indices signify? Sketch (110) and (200) planes in a simple cubic lattice.

(vi) What are Plasmons?

(vii) Why does classical theory fail to explain the temperature dependence of specific heat of solids?

(viii) Differentiate between Type I and Type II superconductors.

3x5

2. (i)Explain Ewald's constuction and derive vector form of Bragg's law **2.K.G+ G²=0** (10)

(ii) Prove that direct lattice is the reciprocal of its own reciprocal lattice. (5)

3(i)Derive and discuss the dispersion relation for linear diatomic lattice. (10)

(ii) Discuss the characterstics of acoustic and optical branches. (5)

4 (i) Discuss the lattice heat capacity of solids based on Einstein's model. What are the limitations of this model.

(10)

(ii) How Debye modified the Einstein model to explain $T^3$ behaviour of solids at very low temperatures? (5)

5(i) Distinguish between diamagnetic, paramagnetic and ferromagnetic substances on the basis of magnetic susceptibility and permeability. (5)

(ii)Discuss Weiss theory of ferromagnetism and explain how magnetic susceptibility varies with temperature. (10)

6. (i)Derive Langevin –Debye formula for the temperature dependence of dipolar polarizability and dielectric constant . (10)

(ii) Explain how this formula is used to study molecular structure. (5)

7. (i) What are ferroelectric materials? Explain P-E hysteresis curve using domain theory of Ferroelectric materials. (7)

(ii) Derive London's equations in superconductivity and discuss London penetration depth and its variation with temperature. (8)

(9)

15/5/18

Sl. No. of Q.P.: 6305 A

Unique Paper Code : 2221602

Name of the Paper : Quantum Mechanics and Applications II

Name of the Course : B.Sc. (Hons.) Physics ~~Physics - FYUP (Sem VI)~~

F-10

Semester : VI

Duration : 3 Hours

Maximum Marks : 75



**Instructions for Candidates**

Question *No. 1* is compulsory.

Attempt *five* questions in all.

1. Attempt any five of the following: $5\times3=15$

(a) Express linear momentum, angular momentum & energy in their respective operator forms.

(b) Prove that the eigenvalues of a Hermitian operator are real.

(c) What are linear operators? Explain with examples.

(d) If $[x, p_x] = i\,\hbar$ then show that $[x^n, p_x] = n\,i\,\hbar\,x^{n-1}$.

(e) List all the states in an *L-S* coupling between two electrons described as *2p* and *2d.*

(f) Discuss the implications of *uncertainty relation* between momentum and position.

(g) Give the values of *n, l* and *j* for the following states written in the spectral notation $2P_{3/2}$, $3D_{5/2}$, $3S_{1/2}$.

2. (a) Let us consider the following matrix operator

$$H = \begin{pmatrix} 2 & i & 0 \\ -i & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Show that the H-matrix is Hermitian and $|\lambda\rangle = \begin{pmatrix} i \\ 7i \\ -2 \end{pmatrix}$ is not its eigenvector.

(b) State, explain and prove the Ehrenfest Theorem

6,9

3. (a) Discuss the physical interpretation of Expectation Values.

(b) Solve the Harmonic Oscillator for its energy eigenvalues using the ladder operator method.

5,10

4. (a) What do you understand by direct product of *ket*-vectors. Explain with examples.

(b) What do you understand by Identical Particles? Differentiate between 'distinguishable' and 'indistinguishable' identical particles.
Show that particles obeying Pauli's exclusion principle can be represented only by Anti-symmetric wavefunctions.
Construct expressions for Anti-symmetric & Symmetric wavefunctions for a system of N-particle Fermions & Bosons respectively using their individual wavefunctions.

4,11

5. (a) Explain, why $L_x, L_y$ & $L_z$ can not have simultaneous eigenstates.

(b) Define the Ladder operators for Angular Momentum and hence show that

$$L_+|l,m> = [(l-m)(l+m+1)]^{\frac{1}{2}}\hbar\,|l,m+1>$$

$$L_-|l,m> = [(l+m)(l-m+1)]^{\frac{1}{2}}\hbar\,|l,m-1>$$

Where $|l,m>$ are the simultaneous eigenstates of $L^2 and\ L_z$ .

3,12

6. (a) Explain the Normal Zeeman effects with examples.

(b) Derive the expression for the Spin Orbit interaction energy and discuss how it explains Fine structure.

6,9

7. (a) State and prove the Variation method for estimating the ground state energy of a quantum mechanical system.

(b) Estimate the ground state energy of the Helium atom using Variation method.

6,9

(8)

SET-A'

17/5/18

Sl-No. of Q.P: 6307



17/5

F-10

Unique Paper Code : 2221604

Name of the Paper : Statistical ~~Physics~~ Mechanics

Name of the Course : B.Sc. (H) Physics

Semester : VI

Duration :3 Hours

Maximum Marks:75

*(Write your Roll No. on the top immediately on receipt of this question paper)*

Attempt five questions in all

Question No. 1 is compulsory.

All questions carry equal marks.

1. Attempt any five of the following: $5\times3=15$

(a) How is Bose gas different from Fermi gas at absolute zero Kelvin?

(b) Calculate the number of modes in a chamber of volume 1 $m^3$ in the frequency range $0.60\times10^{14}$ Hz to $0.61\times10^{14}$ Hz

(c) Explain why negative temperature can exist in a system having two energy levels?

(d) Define Fermi energy. Show graphically its variation with temperature?

(e) Explain the conditions under which Bose-Einstein and Fermi-Dirac distributions approach the Maxwell Boltzmann distribution. Show the variations graphically.

(f) Calculate the partition function, energy and specific heat for a classical system of N particles and three energy levels 0, $\varepsilon$, $2\varepsilon$

(g) If two black bodies have their peak radiations corresponding to violet and red respectively, which of the two is at a higher temperature? Explain through relevant expression.

2. (a) Establish the relation between entropy and thermodynamic probability and show that the constant occurring in the relation is the Boltzmann's constant.

(b) What is the significance of partition function? Calculate the partition function for an ideal mono-atomic gas. Using the partition function obtain the expression for energy, entropy and pressure. (6,9)

(4)

This question paper contains 4 printed pages] 10/5/18

Roll No.

S. No. of Question Paper : **6685**

Unique Paper Code : **32221601** **HC**

Name of the Paper : **Electromagnetic Theory**

Name of the Course : **B.Sc. (H) Physics-CBCS**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

Attempt total **5** questions.

*All* questions carry equal marks.

Question No. **1** is compulsory

Scientific calculator is allowed.

1. Answer any *five* of the following questions : 5×3=15

(*a*) Write the boundary conditions satisfied by the electric and magnetic field vectors at the boundary of the two dielectrics.

(*b*) Explain the terms loss tangent and intrinsic impedance.

(c) Explain why in high frequency circuits current flows only on the surface of conductors.

(d) What do you understand by homogenous and isotropic medium ?

(e) A parallel polarized wave propagates from air into dielectric at Brewster angle of 75°. Find relative permittivity.

(f) For an optical fibre with refractive index of the core 1.47 and of its cladding 1.46. Calculate the pulse dispersion per kilometre.

(g) Given E = $E_0 \sin(\omega t - \beta z)\hat{a}_y$ V/m in free space. Find H.

(h) Write constitutive relations in electrodynamics.

2. (a) State and prove Poynting's theorem. 2,5

(b) What are Lorentz and Coulomb gauges ? Show that Lorentz transformation remains invariant if gauge function $\phi$ satisfies :

$$\nabla^2\phi - \mu_0 \in_0 \frac{\partial^2\phi}{\partial t^2} = 0.$$ 2,6



3. (a) Derive em wave equation satisfied by E field using Maxwell's equation in an isotropic, linear and homogenous dielectric material (no free charges or free currents).

(b) Show that the em waves are transverve in nature.

(c) Calculate the characteristic impedance of the medium.

(d) Show that the energy is equally shared between the electric and magnetic fields in free space. 4,4,3,4

4. (a) Derive Fresnel's formula for the case of propagation of em waves in an anisotropic medium. 9

(b) Show that critical frequency for the propagation of em waves in plasma is $f_c = 9\sqrt{n_0}$, where $n_0$ is number of electrons/m$^3$. 6

5. (a) Derive Fresnel's relations for reflection and refraction of plane em waves at an interface between two dielectric media when electric vector of the incident wave is parallel to the plane of incidence. Also find the expressions for R and T. 8,4

(b) Find the expression for the Brewster's angle at which the reflected wave is completely extinguished. 3

6. (*a*) State Biot's laws for rotatory polarization.

(*b*) Using the Fresnel's theory of optical rotation, obtain the formula for the angle of rotation of plane of vibration.

(*c*) Discuss the construction and working of Laurent's half shade device. 4,6,5

7. (*a*) Determine the change of phase in the reflected ray when it suffers a total internal reflection for the case when E is parallel to the plane of incidence. 5

(*b*) What is step index optical fibre ? Derive relation between the numerical aperture and the angle of acceptance. 6

(*c*) An optical fibre has a core of refractive index 1.48 and cladding of refractive index 1.46, calculate the acceptance angle and the numerical aperture of the fibre. 4

8. (*a*) For the case of propagation of em waves in conducting medium, derive an expression for complex intrinsic impedance and the ratio $u_e/u_m$, where $u_e$ is electric energy density and $u_m$ is magnetic energy density. 8,4

(*b*) What is the significance of the equation : 3

$$\vec{\nabla} \cdot \vec{B} = 0$$

## Section D

8. (a) Given that the chemical potential for a strongly degenerate Fermi gas ($0<T<<T_F$) is :

$$\mu(T) \approx \epsilon_F \left[1 - \frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2\right]$$

Obtain the expressions for the internal energy, pressure, entropy and specific heat capacity. 12

(b) The Fermi energy of metal A is 8.4 eV. Find its value for metal B if the free electron density in metal B is 27 times that in metal A. 3

9. (a) Give two typical characteristics of a white dwarf star. Show that electron gas inside a white dwarf star is strongly degenerate and relativistic. 5

(b) Obtain an expression for the mass-radius relationship for a white dwarf star and hence discuss the importance of Chandrasekhar mass limit. 10

2000



May 2018

*This question paper contains 4 printed pages.*

*Your Roll No.* ..............................

| | | |
|---|---|---|
| *S. No. of Paper* | : **6686** | **HC** |
| *Unique Paper Code* | : **32221602** | |
| *Name of the Paper* | : **Statistical Mechanics** | |
| *Name of the Course* | : **B.Sc. (Hons.) Physics** | |
| *Semester* | : **VI** | |
| *Duration* | : **3 hours** | |
| *Maximum Marks* | : **75** | |

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Attempt **five** questions in all.*
*Question No. **1** is compulsory. Answer **one** question from each Section.*
*Symbols have their usual meanings.*

1. Answer any *five* of the following:

(a) Write two properties of photons which make them different from other bosons.

(b) Derive Stefan's law using thermodynamics.

(c) What is the significance of partition function in statistical mechanics?

(d) List three characteristics of liquid Helium at low temperature.

(e) What do you mean by ultraviolet catastrophe? Explain with the help of a diagram.

(f) Let $\Omega_{MB}$, $\Omega_{BE}$ and $\Omega_{FD}$ be the number of ways in which 3 particles can be distributed in 3 energy states according to M-B, B-E and F-D statistics respectively. Find $\Omega_{MB}:\Omega_{BE}:\Omega_{FD}$.

(g) Calculate the Fermi energy of electrons in silver at absolute zero. Given electron density $= 5.86 \times 10^{28}$ $m^{-3}$.

(h) Give two examples where we can use equipartition of energy theorem. $5 \times 3 = 15$

### Section A

2. (a) Establish the relation between entropy and thermodynamic probability. Show that the constant occurring in the relation is the Boltzmann constant. 7

   (b) A Maxwell-Boltzmann system consisting of 4 particles has a total energy $5\epsilon$. The permitted energy levels are equally spaced with energies $0, \epsilon, 2\epsilon, 3\epsilon$. (i) Write all the possible macrostates, (ii) Determine the thermodynamic probability of each macrostate. 8

3. (a) Discuss the concept of negative temperature from the statistical point of view. Show that it is possible to attain a state of negative temperature for a system of paramagnetic dipoles subjected to an external magnetic field induction ***B***. 10

   (b) Differentiate between 100K and −100K on the basis of entropy-energy diagram. 2

   (c) Consider a system of 13 classical particles distributed initially in three energy states of energies $\varepsilon_1=0$, $\varepsilon_2=2\varepsilon$, $\varepsilon_3=4\varepsilon$, such that $n_1=6$, $n_2=4$, $n_3=3$. If $\delta n_3=-2$, find $\delta n_1$ and $\delta n_2$ and the new distribution of the particles. 3

### Section B



4. (a) Show that the number of modes of vibrations per unit volume of an enclosure in the frequency range $\nu$ to $\nu+d\nu$ is given by $N_\nu d\nu = \frac{8\pi\nu^2 d\nu}{c^3}$ and hence derive Rayleigh-Jeans formula for blackbody radiation. 12

   (b) A black body is placed inside an evacuated chamber maintained at temperature 27°C. If the surface area of the black body is 500 cm$^2$, find energy radiated per time per unit area from the black body when its temperature is 127°C. 3

5. (a) Derive Saha's ionization formula stating its basic assumptions, and highlight its two applications. 10

   (b) Show that the pressure exerted by a diffuse radiation is given by $p=\frac{u}{3}$, where u is the radiation density. 5

### Section C

6. (a) Give the Bose derivation of Planck's radiation law. 7

   (b) Derive the expression for entropy ($S$) and specific heat capacity ($C_v$) for photon gas and hence show that $C_v=3S$. 8

7. (a) What is Bose-Einstein condensation and how is it different from ordinary condensation? Derive the expression for the temperature ($T_c$) at which the Bose-Einstein condensation sets in. 10

   (b) Obtain the expressions for fractions of degenerate Bosons in the ground state ($N_o/N$) and in the excited state ($N_{exc}/N$) as functions of temperature $T$. Plot them. 5

Pending

(6)

14/5/18



[This question paper contains 4 printed pages]

**Your Roll No.** : ....................

**Sl. No. of Q. Paper** : **9496-A** **HC**

Unique Paper Code : 32227613

Name of the Course : **B.Sc.(Hons.) Physics-DSE-4**

Name of the Paper : Communication Systems

Semester : VI

***Time : 3 Hours*** ***Maximum Marks : 75***

**Instructions for Candidates :**

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **five** questions in **all**.

(c) Question NO.1 is compulsory.

(d) Symbols have their usual meaning.

**1.** Attempt any **five** questions from the following : $3\times5=15$

(a) What is angle modulation ? What are the two types of angle modulation ?

P.T.O.

(b) An Antenna has an impedance of 40 Ω. An unmodulated Amplitude Modulated (AM) signal produces a current of 4.8 A. The modulation is 90 percent. Calculate (a) the carrier power (b) the total power and (c) the side band power.

(c) Describe the process to demodulate the Pulse Amplitude Modulated (PAM) wave using an appropriate diagram.

(d) Describe the Quantization process.

(e) Write advantages and disadvantages of geostationary orbits.

(f) Draw the architectural block diagram of mobile communication system.

**2.** (a) What is Modulation ? Distinguish between amplitude and frequency modulation. Calculate the power developed by an AM wave in a load of 100 Ω when the peak voltage of the carrier is 100 volts and the modulation factor is 0.4. 7

(b) Explain the working of a super-heterodyne receiver with the help of a block diagram. 8





**3.** (a) How Frequency Modulation (FM) can be achieved by Voltage Controlled Oscillator (VCO) ? Discuss the demodulation of FM wave using slope detector. 10

(b) Explain the filter method of Single Side Band (SSB) generation. 5

**4.** What are the various pulse modulation schemes. Explain and compare these schemes. Explain Frequency Shift Keying (FSK). 15

**5.** (a) Explain and compare the various pulse modulation schemes. 10

(b) Describe Sampling Theorem. State its relevance towards Pulse modulation. 5

**6.** Write short notes on any **three** of the following : 15

(a) Geosynchronous satellite

(b) Satellite Transponders

(c) Uplink model of satellite link

(d) Downlink model of satellite link

**7.** (a) Explain the structure of the mobile phone cellular network. What is meant by cell splitting and cell sectoring ? 7

(b) What is the difference between GSM and CDMA ? Write a short note on GPS navigation system. 8

(7)

[This question paper contains 6 printed pages.]

Your Roll No. 12/5/18

**Sr. No. of Question Paper : 9502** **HC**

Unique Paper Code : 32227626

Name of the Paper : Classical Dynamics

Name of the Course : **B.Sc. (Hons.) Physics – DSE-3**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all including Question **1** which is compulsory.
3. Attempt at least **one** question from Section **A** and **B.**

1. Attempt any **five** of the following : (3×5=15)

(a) Show that $\dot{\gamma} = \frac{\gamma^3}{c^2}\vec{u}\cdot\vec{a}$

where the Lorentz factor $\gamma = (1 - u^2/c^2)^{-1/2}$ and $\vec{a} = \dot{\vec{u}}$.

(b) Find the Lagrange equation of motion of one-dimensional simple harmonic oscillator.

(c) Show that in calculating the relative proper velocity, Lorentz factors multiply when coordinate velocities add.

(d) Show that $k^\mu = (\omega/c\ ,\ \mathbf{k})$, where $\omega$ is frequency and $\mathbf{k}$ is wave-vector of a wave, is a 4-vector.

(e) If an object has speed $v < c$ in one inertial frame, then using 4-vectors or otherwise prove that $v < c$ in all other inertial frames.

(f) Consider three-dimensional motion in cartesian coordinates (x, y, z) in a one-dimensional potential, say V(x). Identify the conserved quantities.

(g) If the Lagrangian does not depend explicitly on the time, then the Hamiltonian is a constant of the motion.

(h) Find the norm of 4-velocity of a photon moving along x-axis.

## SECTION A

2. (a) Derive Euler-Lagrange equations from Hamilton's principle. (8)

(b) Find the Hamiltonian corresponding to the Lagrangian

$$L = a\,\dot{x}^2 + b\,\dot{y}^2 - k\,x\,y$$

where a, b, k are constants. (7)



3. (a) A particular mechanical system depends on two coordinates $a$ and $b$ and has kinetic energy $T = \dot{a}^2 b^2 + 2\dot{b}^2$, and potential energy $V = a^2 - b^2$. Determine the Lagrangian of this system and derive its equations of motion. (8)

(b) A beam of positive ions (each having mass m and charge q) passes through uniform crossed electric and magnetic fields : $\vec{E} = E_0\,\hat{y}$ and $\vec{B} = B_0\,\hat{z}$. Find the speed of those ions which pass undeviated. (3)

(c) If electric field is switched off then the undeviated ions (in part 3 (b)) move in a circular path of radius R in the presence of magnetic field alone. Find R and hence show that $\frac{q}{m} = \frac{E_0}{R\,B_0^2}$. (2,2)

4. (a) Using Lagrangian or Hamiltonian approach, prove that if particle of mass m moves under the action of central force field then its angular momentum and energy are conserved. (9)

(b) A particle of mass $m$ moves along x-axis under the influence of potential energy

$V(x) = -(kx)e^{-(\beta x)}$, where k and $\beta$ are constants.

Find the equilibrium position and frequency of oscillation. (6)

5. (a) Compare Newtonian mechanics with Lagrangian mechanics. Mention at least three points. (6)

(b) Consider the flow of a viscous fluid (viscosity n) in a channel. The channel has a width in the *y*-direction of *a* and length *L*. There is a pressure drop $\Delta p$ along the length of the channel, so that the constant pressure gradient is generated. Assuming the flow to be steady, $\partial v/\partial t = 0$ and the no-slip boundary condition at the top and bottom edges of the channel, show that the volume of fluid which passes a cross section of the channel per unit time is

$$Q = \frac{\pi a^4 \Delta p}{8 n L} \qquad (9)$$

## SECTION B

6. (a) Explain the phenomenon of Time Dilation (assume clock is in S' frame) using Minkowski space - time diagram. (7)



(b) A particle of rest mass $m_o$ describes a parabolic trajectory $x = at$, $y = bt^2$, $z = 0$ in an inertial frame S. Find the four velocity components. Also find the norm of the four velocity. (8)

7. (a) An inertial frame S' is moving to the right relative to S at a speed 3c/5, and another inertial frame S" is moving to the right relative to S at a speed c/3. Using 4-vectors or space-time diagram, find the velocity of S" relative to the frame S'. (8)

(b) Prove that 4-velocity is perpendicular to 4-acceleration vector. (7)

8. (a) Using four-vector approach derive the expressions for Relativistic Longitudinal and Transverse Doppler effect. (10)

(b) Show that the phase of an electromagnetic wave is Lorentz invariant. (5)

9. (a) Two rods having the same length $L_0$ move lengthwise towards each other parallel to a common axis with the same velocity v relative to the Laboratory frame. Using 4-vectors, show that

$$L = \frac{(1-\beta^2)}{(1+\beta^2)} L_0$$

is the length of each rod in the frame fixed to the other rod where $\beta = v/c$. (7)

(b) The space and time coordinates of two events as measured in a frame S are as follows :

Event 1: $x_1 = x_o$, $t_1 = x_o/c$ ($y_1 = 0$, $z_1 = 0$),

Event 2: $x_2 = 2x_o$, $t_2 = x_o/2c$ ($y_2 = 0$, $z_2 = 0$).

(i) There exists a frame in which these events occur at the same time. Find the velocity of this frame with respect to S.

(ii) What is the value of t at which both events occur in the new frame? (8)

⑧ Portions 19/5/13

Sl. No. of Q.P. 6306

Name of Course : B.Sc. (Honours) Physics (~~FYUP~~)

Semester
~~Scheme/Mode of Examination~~ : VI ~~Semester~~

Name of Paper : Mathematical Physics-IV

F-10

Unique Paper Code : 2221603

Maximum Marks : 75

Time allowed : 3 Hours



Answer a **total** of 5 questions.
**Question No. 1 is compulsory.**
**Answer Two (2) questions from Section A and Two (2) questions from Section B.**
Scientific calculator is allowed.

1. Answer **any five** of the following questions. (3 x 5 = 15)

(a) Let $V = \mathbf{R}^3$. Show that $W = \{(a, b, c): a \geq 0\}$ is not a subspace of V.

(b) Find the dimension of the vector space spanned by (1, -2, 3, -1) and (1, 1, -2, 3)

(c) Is the transformation $T: \mathbf{R}^2 \rightarrow \mathbf{R}$ defined by $T(x, y) = xy$, a linear transformation?

(d) Prove that the eigenvalues of a unitary matrix are unimodular.

(e) Show that every second order tensor can be expressed as the sum of a symmetric and a skew symmetric tensor.

(f) If the metric is given by $ds^2 = 5(dx^1)^2 + 4(dx^2)^2 - 3(dx^3)^2 + 4\,dx^1dx^2 - 6\,dx^2dx^3$, evaluate g.

(g) State the variational principle for the calculus of variation.

(h) What are critical points of a differentiable multi variate function of real variables?

## SECTION-A

*(Answer any two questions from this section)*

2. (a) Let V be the vector space of polynomials of degree $\leq 3$ over **R**. determine whether u, v, $w \in V$ are independent or dependent where:

$$u = t^3 - 3t^2 + 5t + 1,\ v = t^3 - t^2 + 8t + 2,\ w = 2t^3 - 4t^2 + 9t + 5 \qquad (8)$$

(b) Find the matrix representation of the operator T on $\mathbf{R}^2$ relative to the basis $\{f_1 = (1, 3), f_2 = (2, 5)\}$ where T is defined as $T(x, y) = (3x - 4y, x + 5y)$ (7)

3. (a) Let T: $\mathbf{R}^3 \to \mathbf{R}^3$ be the linear mapping defined by

$$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$$

Find a basis and the dimension of the ~~(i)~~ image U of T, ~~(ii) kernel W of T~~ (10)

(b) Show that a matrix A and its transpose $A^t$ have the same characteristic polynomial. (5)

4. (a) Suppose T: $\mathbf{R}^2 \to \mathbf{R}^2$ is an operator defined by T (x, y) = (y, x). Find all eigenvalues and a basis for each eigenspace. (8)

(b) Prove that similar matrices A and B have the same eigenvalues. If **X** is an eigenvector A, then find the eigenvector of B. (4, 3)

## SECTION-B

*(Answer any two questions from this section)*

5. (a) Prove that the Moment of Inertia is given by

$$I_{ij} = \Sigma m(x_k x_k \delta_{ij} - x_i x_j),$$

and write it down in the matrix form. (10)

(b) Show that gradient of a scalar function is a tensor of order one. (5)

6. (a) Using tensors, prove the identity

div (**U** X **V**) = **V**.curl **U** – **U**.curl **V** ~~(7)~~ (5)

(b) Consider the function g(x, y, z) = x – 2y + 5z. Using the method of Lagrange multipliers, find the maximum and minimum values of the function on a sphere $x^2 + y^2 + z^2 = 30$. ~~(8)~~ (10)

7. (a) Let us consider light passing from one medium with index of refraction $n_1$ into another medium with index of refraction $n_2$. Using Fermat's principle to minimize time, derive the Snell's law of refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$. (8)

(b) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (7)